

# Short Papers

## Improved Planar Spiral Transformer Theory Applied to a Miniature Lumped Element Quadrature Hybrid

J. Hogerheiden, M. Ciminera, and G. Jue

**Abstract**— In this paper, an improved method of determining the primary-to-secondary coupling capacitance for planar spiral transformers (PST's) is presented, which enhances previous work. A more general monolithic microwave integrated circuit (MMIC) compatible lumped element multisection model is also presented based on symmetric-width uniformly coupled transmission lines. These techniques were developed to design a 90° hybrid as a MMIC with a center frequency of 2.5 GHz. The design was frequency scaled to 0.5 GHz and fabricated in the microwave integrated circuit (MIC) for verification. Producibility is enhanced and coupling is effectively increased with the novel use of series capacitors which cancel some of the self-inductance of the transformers. Measured results are presented for both a quadrature hybrid and the individual PST used in the quadrature hybrid. The measured results show excellent agreement with the computer models.

### I. INTRODUCTION

Lange couplers [1] are planar devices commonly used in monolithic integrated microwave circuits (MMIC's) at higher microwave frequencies. At frequencies below 3.0 GHz, lengths approach 1 cm on GaAs which is expensive to fabricate. A lumped element implementation can solve this problem. Similar to the way a homogeneous or inhomogeneous transmission line can be modeled with lumped elements [2], [3], a coupled line section can be modeled as an equivalent lumped element circuit.

The planar spiral transformer (PST) is used to realize the lumped element coupler as a planar structure compatible with MMIC technology. The PST structure consists of a pair of tightly coupled lines spiraled in a manner similar to the planar spiral inductors commonly used in MMIC's [4], [5].

### II. LUMPED ELEMENT COUPLER MODEL

Lumped element coupler models using PST's have been described in the literature [6], [7], but the equations are not general. In this section, values for lumped element coupled sections of any electrical length are presented. A 90° coupler can be realized by cascading as many sections of shorter couplers as desired. For instance, two 45° sections or three 30° sections can be cascaded to achieve 90°. Two 45° sections more closely approximates a pair of coupled lines than one 90° section; however, little is gained with more than three sections.

Fig. 1(a) shows the parameters of a distributed coupled-line section, and Fig. 1(b) shows the general lumped element model for a coupled

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J. Hogerheiden is with the American Nucleonics Corporation, Westlake Village, CA 91359 USA.

M. Ciminera and G. Jue are with the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 USA.

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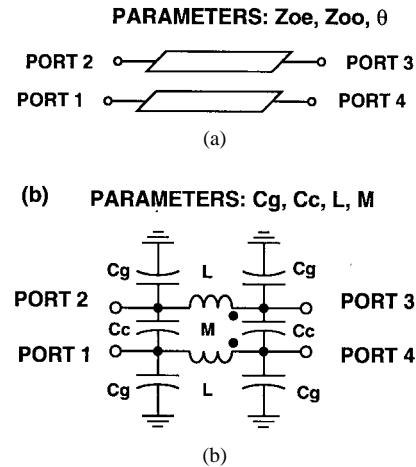


Fig. 1. Coupled transmission line models. (a) Distributed elements. (b) Lumped elements.

line section. Starting from the characteristics of the distributed coupled line section shown in Fig. 1(a), the values for the lumped element model in Fig. 1(b) can be derived by equating the two circuits.

The values for  $L$ ,  $M$ ,  $C_g$ , and  $C_c$  in terms of  $Z_{oe}$ ,  $Z_{oo}$ , and  $\theta$  are as follows:

$$L = \frac{(Z_{oe} + Z_{oo}) \sin \theta}{4\pi f} \quad C_g = \frac{\tan(\theta/2)}{Z_{oe} 2\pi f}$$

$$M = \frac{(Z_{oe} - Z_{oo}) \sin \theta}{4\pi f} \quad C_c = \left( \frac{1}{Z_{oo}} - \frac{1}{Z_{oe}} \right) \frac{\tan(\theta/2)}{4\pi f}.$$

The values of the lumped element model are given in terms of coupled line parameters in a homogeneous medium. Once the lumped elements are determined, any structure which yields the correct lumped element values may be used, whether or not homogenous.

A theoretical lumped element coupler model [shown in Fig. 2(a)] which covers a 3:1 bandwidth using only two 45° sections requires a midband coupling of 2.2 dB. The required coupling factor ( $M/L$ ) for the coupled inductors in this symmetric model is large (0.777). The addition of series capacitors [shown in Fig. 2(b)] effectively reduces the self-inductance of the coupled inductors, allowing greater gap width for a given coupling factor, and a lower  $M/L$  ratio of 0.733. The amplitude and phase balance of the circuits shown in Fig. 2(a) and (b) are very close. The isolation of the circuit in Fig. 2(b) is less than that of Fig. 2(a), but still greater than 20 dB.

### III. PLANAR SPIRAL TRANSFORMER

The PST analysis was based on work done by Frlan *et al.* [4], [5] in which frequency dependent element values were calculated for a lumped element equivalent circuit. Frlan's lumped element "T" network was replaced by an equivalent "Pi" network because it allowed the PST model to be incorporated into the hybrid model shown in Fig. 4. Frlan's work showed good agreement with the measured data on PST's used as a two-port network, where one end of both the primary and secondary windings were grounded.

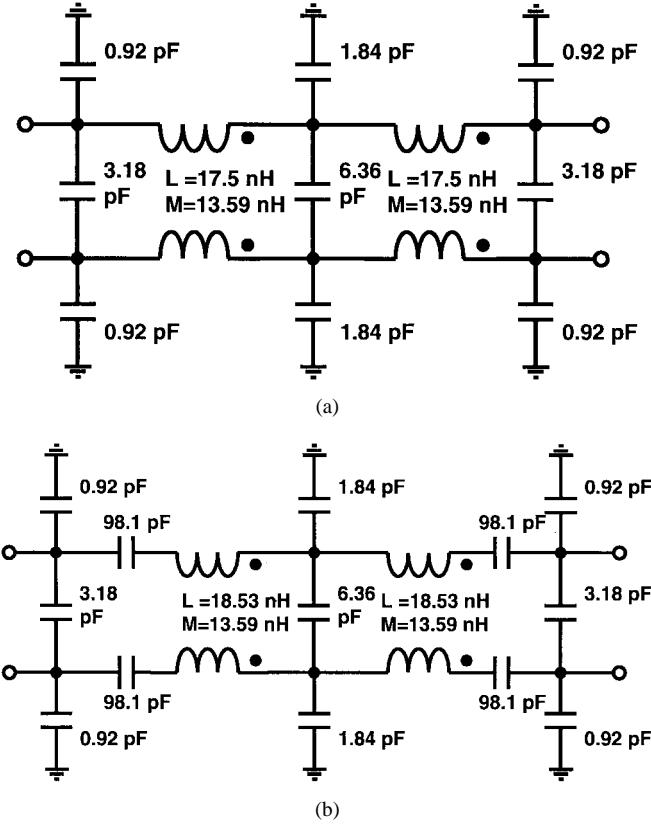


Fig. 2. Theoretical lumped element coupler models. (a) Two-section coupler topology with  $M/L = 0.777$ . (b) Improved two-section coupler topology with  $M/L = 0.733$ .

A  $90^\circ$  hybrid design requires an accurate four-port model, without any of the ports grounded. An in-house computer modeling capability was developed, applying the analysis techniques used by Frlan to analyze PST's used as a four-port network. The model includes the effects of the ground plane and the mutual inductance between all parallel elements. Using the developed modeling capability, the PST required for the hybrid was designed and fabricated.

Original performance predictions based on Frlan's model for grounded PST's did not provide a good match to measured data. The primary source of error in the predicted model was  $C_m$ , the primary-to-secondary capacitance. Frlan's predicted  $C_m$  was too low. Frlan calculates primary-to-secondary capacitance using  $1/2 * (C_o - C_e) * (1 - \cos \Theta)$  to represent the relative phase shift between a pair of segments. Thus, the primary-to-secondary capacitance is equal to zero at dc and increases with frequency.

To provide a more accurate model, a physical basis for the gap capacitance was used. The gap capacitance per-unit length was calculated using Smith's  $C_{\text{gap}} = C_{\text{delta}} = C_{f_{o'}} - C_{f_{e'}}$  [8] and multiplied by the total gap length between primary and secondary. This yielded a *nonfrequency dependent* value of  $C_m$  equal to 1.8 pF, which was 58 times larger at the center frequency than the  $C_m$  calculated using Frlan's method.

New predicted data for the PST was generated using the new value of  $C_m$  with all of the other model element values remaining the same. Very good agreement between the new predicted and measured data is shown in Fig. 3, with  $C_m$  being the only model element different from Frlan's model. Also shown in Fig. 3 is the predicted data using Frlan's technique for calculating  $C_m$ .

Although this analysis and Frlan's both use a constant  $K_{\text{eff}}$  versus frequency, the authors believe this improved technique for calculating

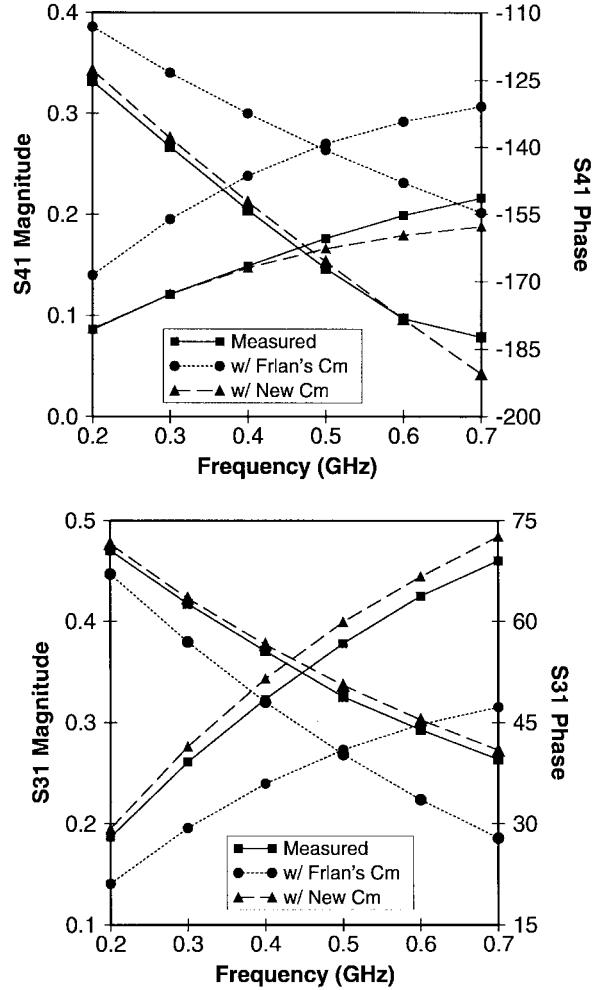


Fig. 3. PST predicted versus measured performance.

$C_m$  is more accurate than Frlan's four-port PST model when used well below resonance.

#### IV. PST QUADRATURE HYBRID

A quadrature hybrid using PST's with series capacitors was developed using the technique described in Section III. This design was done at 0.5 GHz so that it could be built on alumina to verify performance, yet be easily scaled to 1/5 size on GaAs for operation at 2.5 GHz. Parasitics were included in the final model shown in Fig. 4. The shunt capacitors shown in Fig. 2 were realized as radial stubs to eliminate the need for vias. Metal-Insulator-Metal (MIM) capacitors could be used if size were more important than ease of fabrication. The circuit model was adjusted to compensate for the parasitic elements. This reduced the bandwidth, but still met the design requirements.

The PST quadrature hybrid (Fig. 5) was fabricated in the same process as the PST in Section III. The overall size of the PST quadrature hybrid was  $0.92 \times 1.25$  cm at 0.5 GHz. This corresponds to a size of  $3.05 \times 5.08$  mm at the design frequency of 2.5 GHz. The layout was not optimized for reduced size. The use of shunt capacitors requiring vias, rather than radial stubs, would further reduce size. Good correlation between predicted versus measured data for the PST quadrature hybrid is shown in Fig. 6. The predicted data utilized a value of  $C_m$  calculated using [8].

The PST quadrature hybrid was designed for a 2.2 dB maximum coupling and 3:1 bandwidth at 1.8 dB amplitude difference and

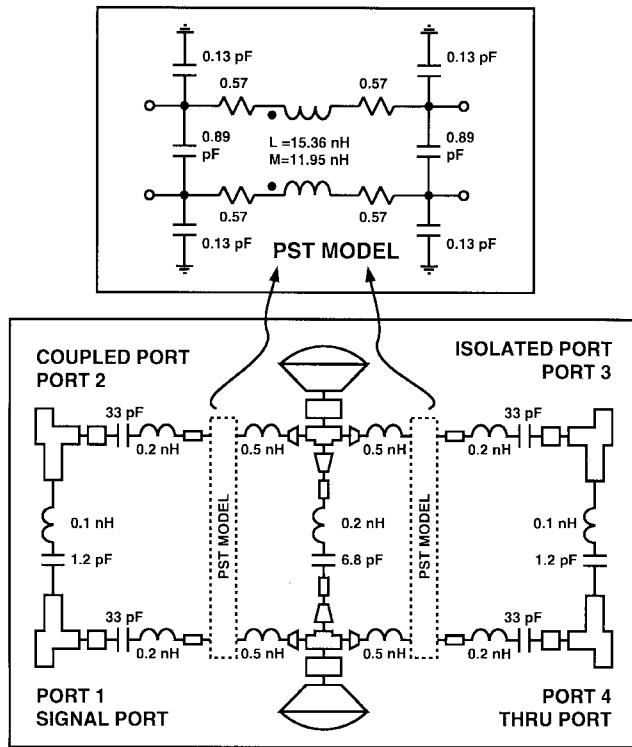


Fig. 4. Model for fabricated hybrid.

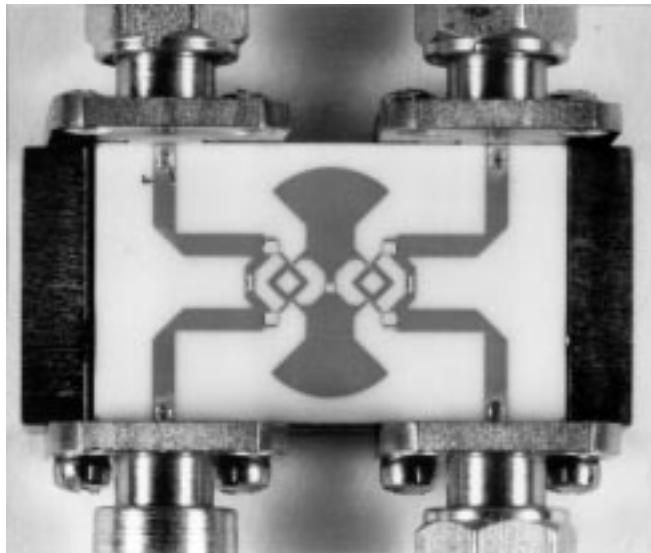


Fig. 5. PST quadrature hybrid photo.

achieved 2.1-dB maximum coupling and 2.8:1 bandwidth at 1.77-dB amplitude difference.

## V. CONCLUSION

Frlan's model for calculating  $C_m$  is inaccurate for transformers used as a four-port network. His work is based on grounded transformers and the value of  $C_m$  is effectively zero at dc and increases with frequency. The authors have demonstrated a more accurate technique for calculating  $C_m$  which agrees well with measured data.

The authors have shown more general design equations for lumped element multisection couplers. In addition, the authors have shown that highly miniaturized broad-band quadrature hybrid couplers are

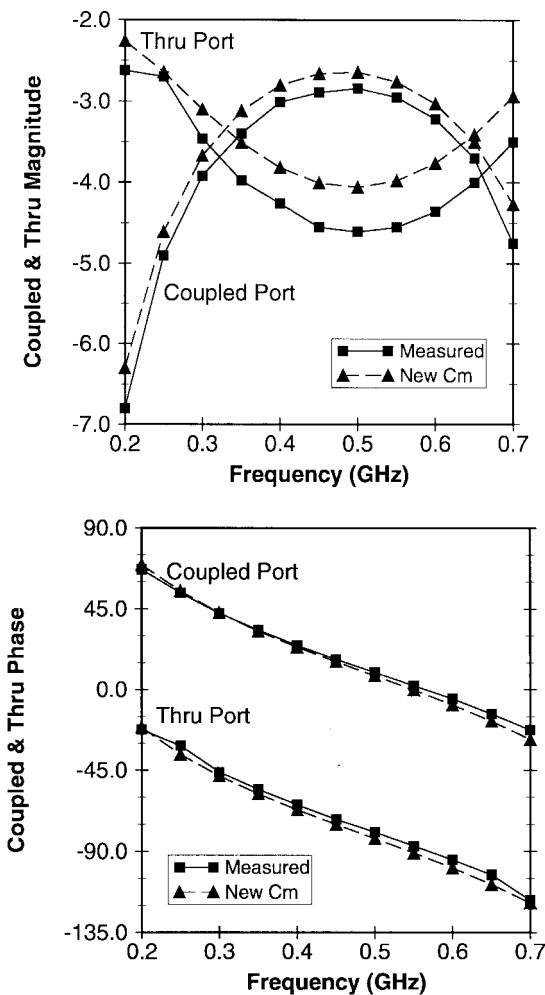


Fig. 6. PST hybrid predicted versus measured performance.

made more realizable using PST's with series capacitors. This design can offer size reduction over Lange couplers and is compatible with MMIC technology, making it a valuable building block for integration into larger MMIC circuit designs.

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